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# Gottfried Haberler's Contributions to the Theory of Index Numbers: A Blueprint for Revealed Preference Theory

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#### Abstract

This paper reviews the neglected contributions to the theory of index numbers in Gottfried Haberler's (1927) habilitation thesis *Der Sinn der Indexzahlen (The Meaning of Index Numbers)*. Haberler was the first to present an argument for why the rates of change of the Paasche and Laspeyres indices can be interpreted as the lower and upper bounds of the actual change in the price level under certain assumptions. He thus provided a rationale, based on economic theory, for taking averages of the Paasche and Laspeyres indices, such as the Fisher index, as approximations to the actual rate of change. His argument is firmly rooted in the subjective value tradition of the Austrian school, but it requires preferences to be held constant. His line of argument provides a blueprint for what later became known as revealed preference theory developed by his student in the 1930s, Paul Samuelson.

*Keywords:* index numbers; price level; revealed preference; Gottfried Haberler; Paul A. Samuelson *JEL-Codes:* B25; C43; E31

### 1 Introduction

The Austrian-born economist Gottfried Haberler (1900-1995) is well known for his contributions to the theories of international trade (Haberler, 1929, 1933, 1950, 1951; Baldwin, 1982; Willett, 1982) and business cycles (Haberler, 1937; Officer, 1982; Boianovsky and Trautwein, 2006). As a professor at Harvard, from 1936 until his retirement in 1971, he was one of the best-established economists who had left the German-speaking world after the rise of the national socialists. Over two generations he has trained some of the leading American economists. Most famously among them was Paul Samuelson who attended Haberler's first lectures at Harvard on international trade in 1936 when he was only 21 years old. In fact, Harold Hitchings Burbank, then chairman of Harvard's Economics Department, recommended to Samuelson in 1935 to postpone taking the class on international trade until "next year when the great young authority, Haberler, would be giving it" (Samuelson, 1992, p. 101).

Over three decades later, when Samuelson was consulted by the Swedish Royal Academy of Science in 1968 to suggest worthy recipients of the first Nobel Memorial Prize in Economics to be awarded in 1969, he listed his former teacher Haberler besides 13 other renowned economists above the age of 60.<sup>1</sup> Haberler was never given the honor his student thought he deserved. On the occasion of Haberler's 90th birthday, Samuelson made his assessment public in a laudatory speech during a reception held at the American Enterprise Institute. He remarked that Haberler's contributions would "qualify him for about two-and-a-half Nobel Prizes in Economics - one for his quantum improvement in trade theory beyond Ricardo's paradigm of labor's comparative advantage, one for his definitive synthesis of business cycle theory, and beyond these his policy wisdoms over a period of six decades" (Samuelson, 1992,

<sup>&</sup>lt;sup>1</sup>Besides Gottfried Haberler, Samuelson listed Frank H. Knight, Alvin H. Hansen, Jacob Viner, Jan Tinbergen, Ragnar Frisch, Bertil Ohlin, Gunnar Myrdal, Roy Harrod, John Hicks, Joan Robinson, Wassily Leontief, Nicholas Kaldor, and Abba Lerner. Six of them eventually won the Prize. Frisch and Tinbergen were the first in 1969. He also listed 12 younger colleagues of which 7 would win it: Lloyd Metzler, James Tobin, Robert Solow, Kenneth Arrow, Franco Modigliani, Maurice Allais, Tjalling Koopmans, Milton Friedman, Erik Lundberg, Ingvar Svennilson, Hermann Wold, and Henri Theil (Düppe and Weintraub, 2014, p. 209-210).

p. 101).

Samuelson was probably tempted to exaggerate on such a special occasion, but there is no question that Haberler had made major contributions. Most economists familiar with his work would immediately agree, but only few appreciate the fact that his significant contributions to economics go far beyond international trade and business cycles. His first main publication was an exploration and critical discussion of the theory of index numbers. He covered this rather technical topic in his habilitation thesis defended at the University of Vienna in the 1920s. The thesis was published shortly afterwards as Der Sinn der Indexzahlen (The Meaning of Index Numbers) (Haberler, 1927). It has never been translated and never reached a broader international audience.<sup>2</sup> In an obituary in *The Washington Post* it has been claimed that his main insights on the theory of index numbers are incorporated in the calculation of modern consumer price indices (WP, 1995), but what exactly were his insights? And are they really incorporated in the calculation of consumer price indices that serve as primary targets of monetary policy around the world? It seems safe to say that at least some important ideas from Haberler's habilitation thesis have been neglected and forgotten. They have never been fully appreciated. The purpose of this paper is to rectify this gap and bring the gist of Haberler's analysis of index numbers back into the academic discourse.

### 2 The Meaning of Index Numbers

In the introduction to *The Meaning of Index Numbers* (MOIN), Haberler (1927, p. III) identified a divide between the "Charybdis of blind belief in numbers and the Scylla of negative skepticism",<sup>3</sup> between which he attempted to reach a nuanced compromise. The main thinkers associated with the opposing camps were, in Haberler's view, Irving Fisher on

 $<sup>^{2}</sup>$ There is a small League of Nations memorandum on fluctuations in the purchasing power of gold that contains some of his findings, but by no means the most significant ones (Haberler, 1931).

<sup>&</sup>lt;sup>3</sup>All direct quotations from MOIN are translated by myself. The original German quotes are given only when translations are not straightforward or when there could be different translations with different meanings.

the side of the blind believers, supported by men like William T. Foster and many others. On the other side of the aisle, amongst the negative skeptics, stood economists like Nicolaas Pierson, Knut Wicksell and Ludwig von Mises. According to Haberler, the skeptics did not gain credibility through their numbers, but through the weight of their voices. The enthusiasts were already in the majority back then. They still are today. If anything has changed, the skeptics seem to be fewer than ever. And they have no prominent spokesmen left.

The vice of the blind believers in index numbers, according to Haberler, is that they do not take time to spell out the meaning of what it actually is that they try to measure. They do not carefully define what the *price level* or the *purchasing power of money* are. They take some price average that satisfies certain formal criteria and run with it. The vice of the skeptics is that they tend to deny the importance of index numbers in many practical applications. We cannot do without some notion of the price level and an estimation of changes in the purchasing power of money, if we want to assess the evolution of real wages or real economic growth over a given historic period, or if we want to compare them between regions. Haberler's habilitation thesis is an attempt to find some middle ground between blind belief and exaggerated skepticism. In spite of seeking nuance, there was at least one American reviewer of the book who thought that Haberler takes part in the group of the skeptics after all (Taylor, 1929), which is not surprising given his close association with the Austrian school and, in particular, Ludwig von Mises and his private seminar that he among many other prominent economists attended regularly (Haberler, 1961; Hülsmann, 2007; Linsbichler, 2022; Caldwell and Klausinger, 2022, pp. 107,143).

In January 1979, Haberler was asked in an interview what the essence of his argument in MOIN was and what he thought about the widespread use of macroeconomic aggregates in Keynesian economics. His reply again shows a desire for balanced interpretation:

Well, that was a technical book on index numbers, and I tried to link up the index number problem with general economic theory, but it did not have much on the particular problem to which you are referring. Now, I hear that Keynesians are criticized for using aggregates. I think that everybody uses aggregates.

Of course, you can overdo it. You can make the aggregates too large, but the economy is simply too complicated to go all the way down in a practical way to the last minute microeconomic distinctions. So I think that criticism of the aggregate approach has been overdone. It is a matter of degree and not either-or. [...]

I did mention in my book on index numbers that it is possible that the price level changes differently for different classes of goods and consumers, but here you have the practical problem that you cannot easily handle such fine distinction. You would not, however want to say that index numbers are useless because you would need an infinite number of index numbers to incorporate all the microeconomic distinctions. That I definitely did not mean to say in my book. (Haberler, 2000)

It is true that Haberler did not reject all attempts to quantify changes in the purchasing power of money. However, he also emphasized one fundamental point right from the beginning: "It turns out [...] that we can only speak of a subjective price level, of a subjective purchasing power of money" (Haberler, 1927, p. v). Firmly based in the subjective theory of value as developed by (Menger, 1871), Haberler proceeds to assess the challenges and limitations of index number calculation both from the mathematical-statistical point of view (MOIN, Part I: pp. 1-69) and the economic point of view (MOIN, Part II: pp. 70-127).

#### 2.1 The Mathematical-Statistical Point of View

The mathematical-statistical part of the book is overall of less interest to us. It contains many basic and well-known considerations concerning different kinds of averages, indices and combinations of them. We will, however, briefly present Haberler's main findings and introduce some of the notations used later on. One of the first questions Haberler addresses is whether an index number that tries to capture an average price change should be constructed as an *average of price ratios* or as a *ratio of price averages*. Here, Haberler (1927, p. 10) refers to Jevons (1884) who argued that an average for prices of different goods, like a ton or iron, on the one hand, and a bushel of wheat, on the other, is meaningless. There is no average price for wheat and iron. They have no connection. However, price ratios for goods over time (or regions) are different as they do not depend on the incommensurable units of the respective goods. The ratio of the iron price in period 2 to the iron price in period 1 can be meaningfully compared to the ratio of the wheat price between the two periods, regardless of their units. Fisher (1922) shares this view. He believes every useful index is an average of price ratios. Haberler agrees, but he points out that, in most accepted cases, an index number can be expressed as either a weighted average of price ratios or a ratio of weighted averages. It is, in fact, both at the same time. Furthermore, such indices can be expressed as a ratio of price sums. And it is this last representation that for Haberler is the most useful. In fact, he argues that

Every price average, even an average of *price ratios* is pointless, if it cannot be understood as the ratio of price sums. Understandable and meaningful are only those index numbers, which can somehow be traced back to a ratio of price sums, and more specifically price sums of one and the same combination of goods. (Haberler, 1927, p. 19)

Let us illustrate these different expressions with the two standard formulas of index number theory: the Laspeyres index  $(\mathcal{L}_I)$  and the Paasche index  $(\mathcal{P}_I)$ . In a two-period scenario, where  $p_1$  and  $p_2$  refer to the unit prices of goods and services in period 1 and 2, and  $q_1$  and  $q_2$  refer to the respective quantities that a customer has bought of these goods in the two periods, we can write both indices as *ratios of weighted price averages*:

$$\mathcal{L}_{I} = \frac{\sum \frac{q_{1}}{\sum q_{1}} p_{2}}{\sum \frac{q_{1}}{\sum q_{1}} p_{1}}; \quad \mathcal{P}_{I} = \frac{\sum \frac{q_{2}}{\sum q_{2}} p_{2}}{\sum \frac{q_{2}}{\sum q_{2}} p_{1}},$$

where the weights for the prices are constructed on the basis of the quantities of period 1,  $q_1/\sum q_1$ , for the Laspeyres index, and on the basis of the quantities of period 2,  $q_2/\sum q_2$ , for the Paasche index.<sup>4</sup>

These two equations can also be rewritten as averages of weighted price ratios:

$$\mathcal{L}_{I} = \sum \frac{p_{1}q_{1}}{(\sum p_{1}q_{1})} \frac{p_{2}}{p_{1}}; \quad \mathcal{P}_{I} = \sum \frac{p_{1}q_{2}}{(\sum p_{1}q_{2})} \frac{p_{2}}{p_{1}};$$

where the weights for the price ratios  $p_2/p_1$  in the Laspeyres index correspond to the expenditure shares in period 1:  $p_1q_1/\sum p_1q_1$ . The weights of the Paasche index replace the quantities of the first period by the quantities of the second:  $p_1q_2/\sum p_1q_2$ .

The easiest and most convenient way of presenting the two indices is typically as *ratios* of price sums:

$$\mathcal{L}_I = \frac{\sum p_2 q_1}{\sum p_1 q_1}; \quad \mathcal{P}_I = \frac{\sum p_2 q_2}{\sum p_1 q_2};$$

In this form the two standard indices can be interpreted easily as each one of them is recognized as a ratio of "price sums of one and the same combination of goods" (p. 19). The Laspeyres index takes the combination of goods of period 1. Its costs are computed at prices of period 2 ( $\sum p_2q_1$ ) and at price of period 1 ( $\sum p_1q_1$ ). The ratio of the two price sums gives the index. The Paasche index takes the combination of goods from period 2 instead. All else remains the same.

The Laspeyres index therefore compares the total price of the bundle of goods chosen in

$$\mathcal{L}_{I} = \frac{\sum_{i=1}^{n} \frac{q_{1i}}{\sum_{j=1}^{n} q_{1j}} p_{2i}}{\sum_{i=1}^{n} \frac{q_{1i}}{\sum_{j=1}^{n} q_{1j}} p_{1i}}.$$

<sup>&</sup>lt;sup>4</sup>We take the same shortcut in the notation that Haberler took in his book by simply leaving out the indices running over the goods' categories. For n categories it should be:

However, the formulas are understandable without the i's and the j's, and this way, the reader gets a bit more of the original flavor of the book. It must have been terribly painful to typeset these equations back in the 1920s. We occasionally correct minor errors in Haberler's exposition without explicitly mentioning them.

period 1. If this bundle would be more expensive in period 2, at prices  $p_2$ , than it was in period 1, at prices  $p_1$ , the index indicates an increase in the price level,  $\mathcal{L}_I > 1$ . The Paasche index compares the total price of the consumption bundle chosen in period 2. If this bundle would have been cheaper in period 1, at prices  $p_1$ , than it is in period 2, the index indicates an increase in the price level  $\mathcal{P}_I > 1$ .

The representation of the indices as ratios of price sums is very convenient for interpretation, but as we have seen above, they can just as well be written as ratios of *weighted* price averages or as averages of *weighted* price ratios. All the action is in the weights.

As Haberler points out these two standard formulas do not give the same results. And there is no good reason to prefer one over the other. The whole quest for the correct index number is a quest for the correct weights and for the correct reference bundle. Should one take the bundle of the first period or that of the second, a compromise between the two, or something completely different? A large portion of the text compares the mathematical properties of these indices and different compromise formulas, the most famous of which is Fisher's "ideal formula" that takes the geometric average of the Laspeyres and the Paasche index. Such a compromise formula, Haberler (1927, pp. 34-35) argues, "is only coherent if one can demonstrate that the standard formulas constitute bounds, that is, they demarcate the space in which the wanted value lies." Haberler will give such a demonstration in the second part of the book. The basic gist of the first part is that the demonstration is not to be found in the formal mathematical properties of the indices, but has to be derived from the underlying substance of what it is we try to measure.

The economist in Haberler clearly comes through in certain passages of the first part already. For example, when he discusses March (1921) who presents arguments for an unweighted average, detached from the quantities actually bought, because what is relevant in assessing the purchasing power of money, is not so much what people *actually* buy, but what they could *potentially* buy. The purchasing power of money, according to this line of reasoning, does not really depend on concrete buying and selling decisions. Haberler (1927, p. 28) gives the analogy of a magnet. Its pull does not depend on actually pulling things towards it. Even if it does not pull anything, the force is still there. But this, as Haberler argues, shakes at the very foundation of the problem of index numbers. In anticipation of the analysis following in the second part of the book, he writes:

If we want to see clearly, we have to try to specify the vague term purchasing power. Without going into a critical discussion of the various views here, which will be done at a suitable point, we can say that we can only make sense of the term purchasing power by somehow linking it to the utility and significance of money for economic subjects. For a follower of the Austrian school of economics, this goes without saying. (Haberler, 1927, p. 29)

In essence, the purchasing power of money is not an objective property like the pull of a magnet. It is rendered meaningful only through the evaluation of economic agents as manifested in concrete buying and selling decisions. If we are searching for the best average or index of prices, we have to look at the problem through the lens of economics. It is not primarily a mathematical-statistical challenge.

This perspective is forcefully illustrated by Haberler's discussion of Walsh (1901) who argued that the geometric average is to be preferred over the arithmetic average when it comes to prices. Haberler (1927, pp. 37-40) picks up an exchange of letters between Galileo Galilei and the mathematician Castelli, on one side of the debate, and a certain Nozzolini, on the other. Walsh refers to this 17th-century debate that is contained in the collected works of Galilei. Walsh ends up siding with the famous polymath to emphasize his own argument.

The question discussed between the Italian gentlemen is the following: Assume there is a horse worth \$100. One person estimates the value of the horse to be \$10, another one thinks it is \$1,000. Who is less wrong? Now, this question is highly relevant for the topic at hand, because it indirectly asks what would the value of the horse have to be, so that both estimates are equally wrong? It is therefore asking for the proper average of the two. Nozzolini believes the arithmetic average to be the appropriate measure. He thinks the value of the horse would have to be \$505 for both estimates to be equally wrong. One overestimates by \$495, the other underestimates by \$495. By contrast, Galilei and Castelli believe that, given the true value of \$100, the two estimates are equally wrong as they stand. One estimation is ten times smaller (\$10), the other is ten times bigger (\$1,000). They are equally wrong in proportion to the true value. So, Galilei and Castelli believe that the geometric mean is the appropriate measure.

Nozzolini tries to support his position by arguing that if the two persons each were to buy a horse that is actually worth \$100 at their estimated prices of \$10 and \$1,000, one would make a loss of \$900, and the other would make a profit of \$90 that hardly compensates the loss. The loss is bigger and hence the high estimate is further away from the truth. Galilei counters that one cannot simply assume actual purchases. Estimations are not about compensating profits and losses. They cannot be quantified in specific units, like money, meters, or pounds. They can only be quantified in relation to the true value. If one person overestimates the height of a hill by 100m and another person underestimates the weight of a cow by 10lbs, one cannot say by these numbers alone what estimate is better. Only in proportion to the true value can the estimates and the errors be compared.

Castelli adds a *reductio ad absurdum* by asking the following question: If one person estimates the value of the horse to be \$200 or even more, what value below \$100 would the other person have to pick to commit the same error in the opposite direction? If one estimate is \$200, the other would have to be \$0. If it is more than \$200, the other would have to be even negative. But a value of \$0 or below is clearly a bigger error. The case is even more obvious, when considering the height of a hill. It is possible that someone thinks a hill is 220m high, although it is only 100m high. But nobody would mistake a hill for a whole in the ground! With the arithmetic mean there is no compensating error in the other direction if the first estimation is 220m. With the geometric mean there is. An estimate of 200m would be equally wrong as an estimate of 50m. As the first estimate approaches infinity, the other would fall to zero, but never below. Walsh (1901, p. 245) takes this example and formulates a generalized conclusion: the arithmetic average is useful, where values are concerned that can vary in both directions without restrictions. However, prices are restricted in one direction. They can fall to zero, but never below. There is no upper bound. They are "one-ended phenomena."<sup>5</sup> Hence, the geometric average is generally preferred, when it comes to prices. Haberler (1927, p. 40) is not convinced:

However, this justification of the geometric mean is inadequate. It is another typical example of how one tries to solve factual economic problems with formal mathematical considerations. The whole derivation is obviously only conclusive if one tacitly assumes that every price increase of one good - if we only calculate with 2 types of goods - must be able to be compensated by a price reduction of the other. In other words: if we assume that there are only 2 types of goods, then any increase in the price of one good, even for more than 100%, must be able to be compensated by a fall in the price of the other good.

In the real world that is not the case. There are prices that, if they increase significantly, cannot be compensated by a fall in just any other price. But this is the underlying assumption in Walsh's discussion. Haberler gives the following example. Imagine that a consumer buys a certain bundle of goods at given market prices. If for half of the goods, that is, a part of the bundle which accounts for 50% of overall expenditure, prices more than double, it is clear that the old bundle is not affordable anymore with the same amount of money, even if, prices of the other goods fall to zero. So it is quite possible that a doubling in certain prices can just not be compensated, even if other prices fall to zero. It is simply not about the physical quantities of goods. If one price doubles and the other one is halved, we can buy twice as much of one good and only half as much of the other, if expenditure shares remain the same. That does not imply that the loss of half the quantity of one good is compensated

<sup>&</sup>lt;sup>5</sup>Haberler (1927, p. 40) refers to Edgeworth who used this expression by quoting Venn.

by the doubling in the quantity of the other. What matters is the subjective value attached to the bundles.

Haberler closes the first part of his thesis by making some very interesting observations concerning prices and quantities of goods in general. He does this in the context of discussing the *factor reversal test* as a quality test for index numbers. It is emphasized in Fisher (1922) who holds that for every price index there is a corresponding quantity index. The multiplication of both corresponds to the ratio of nominal expenditures. For example, a consumer spends overall  $\sum p_1q_1$  in period 1 and  $\sum p_2q_2$  in period 2. The ratio of nominal expenditures is

$$\frac{\sum p_2 q_2}{\sum p_1 q_1}$$

It contains both price and quantity changes. The whole point of constructing a price index is to separate the price change from the quantity change. The product of a price index  $P_{12}$ and a quantity index  $Q_{12}$  should get you back to the ratio of nominal expenditures:

$$P_{12}Q_{12} = \frac{\sum p_2 q_2}{\sum p_1 q_1}.$$

Now, it is easy to see that the Fisher index satisfies the factor reversal test. The Fisher price index is the geometric mean of the Laspeyres and the Paasche index:

$$\mathcal{F}_{PI} = \sqrt{\mathcal{L}_I \mathcal{P}_I} = \sqrt{\frac{\sum p_2 q_1}{\sum p_1 q_1} \frac{\sum p_2 q_2}{\sum p_1 q_2}}$$

The corresponding Fisher quantity index is obtained by reversing the p's and the q's. Hence,

$$\mathcal{F}_{PI}\mathcal{F}_{QI} = \sqrt{\frac{\sum p_2 q_1}{\sum p_1 q_1} \frac{\sum p_2 q_2}{\sum p_1 q_2}} \sqrt{\frac{\sum q_2 p_1}{\sum q_1 p_1} \frac{\sum q_2 p_2}{\sum q_1 p_2}} = \frac{\sum p_2 q_2}{\sum p_1 q_1}.$$

The factor reversal test is passed. Fisher gives two reasons why this is important. First, there is no reason why a formula that can be applied to prices could not also be applied to the quantities. And second, the factor reversal test clearly works for individual price and quantity changes

$$\left(\frac{p_2}{p_1}\right)\left(\frac{q_2}{q_1}\right) = \frac{p_2q_2}{p_1q_1},$$

so it should also work for indices. But only few weighted index numbers actually satisfy this rule. Of course, to every price index we can construct a counterpart that satisfies the rule, but it does not necessarily come from the same formula.

Take, for example, the Laspeyres price index and the corresponding Laspeyres quantity index. They do not pass the test:

$$\frac{\sum p_2 q_1}{\sum p_1 q_1} \frac{\sum q_2 p_1}{\sum q_1 p_1} \neq \frac{\sum p_2 q_2}{\sum p_1 q_1}.$$

However, there is of course a counterpart to the Laspeyres price index, which happens to be the Paasche quantity index, since:

$$\frac{\sum p_2 q_1}{\sum p_1 q_1} \frac{\sum q_2 p_2}{\sum q_1 p_2} = \frac{\sum p_2 q_2}{\sum p_1 q_1}.$$

So the Laspeyres price index finds its counterpart in the Paasche quantity index, just as the Paasche price index finds its counterpart in the Laspeyres quantity index. For Fisher this is a reason to reject both of them and stick to his "ideal formula."

Haberler (1927, p. 64) argues that it is no surprise at all that Fisher emphasized the significance of this rule. The index is supposed to play an important role in Fisher's monetary theory and ought to fit the equation of exchange:  $MV = \sum pq = PQ$ . As pointed out before, Fisher rejected the notion of an average price, so he did not actually believe that  $\sum pq$  could be meaningfully separated into an average price and quantity component PQ. He thought instead in terms of price and quantity ratios, or "relatives" as called them. He believed that indices for the ratios,  $P_{12}$  and  $Q_{12}$ , could be found, such that

$$P_{12}Q_{12} = \frac{\sum p_2 q_2}{\sum p_1 q_1}$$

In this context, Haberler quotes directly from Fisher who refers to US data on the total nominal value of sold quantities of 36 commodities:

Thus we can say with absolute certainty that the total value in 1918 was 223 percent of the total value in 1913. But when we ask how far this increase from 100 to 223 represents increased prices and how far it represents increased quantities, we enter the quagmire of index numbers. We are searching for a formula which, applied to prices, will really measure the increase of the prices, and, applied to quantities, will really measure the increase of the quantities; and such that to make these two results consistent, their product should give the required 223 percent. (Fisher, 1922, p. 75)

The question Fisher asks, which is also emphasized by other index theoreticians like Gini (1925), is this. What index does properly separate price changes from quantity changes? We have seen that the Fisher index does, but Haberler claims that the question is just wrong. It "is based on a fiction" (Haberler, 1927, p. 65). Instead he asks: "Does it make any economic sense at all, is it possible to ask and say which part of the change in total turnover is attributable to price changes and which to quantity changes? Is this a theoretically tenable question?"<sup>6</sup>

Prices and quantities are interdependent entities. Basic price theory tells us that prices are determined by quantities, namely, the quantities demanded and supplied. The clean separation that index theorists try to construct is artificial. It is worth to quote Haberler at some length on this point, where his training in the Austrian tradition really comes through:

<sup>&</sup>lt;sup>6</sup>Haberler notes that this question should not be confused with another question, namely, whether a price change is caused from the money side or the goods side. This is a completely different question, which can be seen in the fact that both causes on the goods side and causes on the money side will change both prices and quantities. He refers in this context to a forthcoming book by Hayek which is most likely his *Monetary Theory and the Trade Cycle* which appeared for the first time in German in 1929 (Hayek, 1933).

Let's take a single good as an example. Its turnover changes from  $p_1q_1$  to  $p_2q_2$ . Can we ask what is to blame? The quantity or the price? The answer should be found in price theory. Price theory says that prices change when supply and demand change, or more precisely, when quantities offered and demanded (which can of course be explained further back) change. When supply and demand shift, the turnover, i.e. the quantity of goods sold, changes and uno actu the price and the amount of money spent; because the latter is nothing other than the mathematical product of the price per unit and the number of these units. The p and the q are factors in this mathematical product pq. However, they are not real factors in the sense that they produce the total turnover together and are causally determining for its changes. It is essentially pointless to ask about the causal share of two factors in the result of a multiplication. In other words: prices and quantities are dependent parts of one and the same real phenomenon, of the same real processes, namely the total turnover, which ultimately - like everything in the economy - is nothing other than an epitome of human actions, of acts of exchange. And its size is determined by everything that determines these actions of the economic subjects [...]. Among other things, the quantities of goods in the possession of the economic subjects are co-determining (but these are the quantities available for exchange and do not necessarily have to be the actual quantities coming into exchange, - the q occurring in the index formulas!) If the available quantity increases, the supply increases and the price decreases and vice versa. Thus both, the q and the p, change as a result of a change in the available quantity, and one cannot therefore determine their influence |...| by first allowing one factor, the price, to vary and then the other. (Haberler, 1927, p. 67)

The factor reversal rule that presupposes a strict separation of both factors is thus rejected by Haberler on the basis of these considerations. To him it sounds "preposterous" (p. 69) when Fisher (1922, p. 228), after having computed and compared 13 selected indices including his "ideal formula" (no. 353) for US data from 1913 to 1918, claims that

The largest error in a single index number is that for 1917 relatively to 1913. That is the index number has an error of .128 per cent, or about one part in 800, or about one eighth of one per cent. We may, therefore, be assured that Formula 353, being certainly more accurate, if that be possible, than at least most of the other 12, is able correctly to measure the general trend of the 36 dispersing price relatives or quantity relatives within less than one eighth of one per cent! That is, the error in, say, Formula 353, probably seldom reaches one part in 800, or a hand's breadth on the top of Washington Monument, or less than three ounces on a man's weight, or a cent added to an \$8 expense.

Fisher looks at the differences between his calculated indices to reach this optimistic error assessment. All of this presupposes that these index numbers are by and large shooting in the right direction. But what is the reason to believe that? Haberler's main conclusion is that we cannot by means of mathematical-statistical considerations get to the bottom of things. The application of index numbers to prices and quantities that are simultaneously determined on markets requires an economic point view.

#### 2.2 The Economic Point of View

Haberler begins the second part of his thesis by introducing the idea of a "collective concept" (*Kollektivbegriff*). Such concepts are widespread in the social sciences. They subsume an indefinite number of individual actions. They summarize "an inter-individual diversity of acts" (*eine interindividuelle Aktmannigfaltigkeit*) (Haberler, 1927, p. 72). Examples of collective concepts are the interest rate, the balance of trade, rents, wages, as well as the exchange value of money or its purchasing power. According to Haberler even a single market price of a good or service is a collective concept. When we say that the price of some good in

some place at some point in time was p, we summarize an indefinite number of exchanges of a certain type. The danger of these collective concepts is that they are often seen as one, or as a unit, rather than a collective of individual actions. This has generated "infinite damage" in economics and "a tremendous amount of useless quarreling about pseudo-problems" (p. 73). An example according to Haberler is the entire discussion surrounding the dichotomy of inner and outer exchange value of money.<sup>7</sup> The value of money and the price level are collective concepts that are prone to such misinterpretations. Before continuing his analysis, Haberler (1927, p. 73) therefore declares:

In the following, we always want to remain aware that these terms are collective concepts, i.e. that they refer to a majority of economic acts, and we want to fall back as often as possible, at least in difficult situations, on the final elements of all economic activity, to human actions, as it were as a test, so that we are not misled by collectivist mystifications.

Haberler moves on to emphasize the difference between *monism* and *pluralism* in the theory of index numbers. The monists regard the price level and the purchasing power of money as an objectively determinable magnitude. Walsh (1901), for example, believes that there exists an objective price level that is the same for all agents throughout the economy, and he believes that the exchange value of goods is an inherent property of the goods themselves. He is a "pure monist" (Haberler, 1927, p. 75). Walsh believes that there is one homogeneous economy-wide exchange value of money - an objective exchange value *as such*. An index that captures it, is suitable to serve all kinds of purposes. A similar view was expressed by Fisher (1911), although he later transitioned to a more pluralist perspective (Fisher, 1922).

Haberler (1927, p. 76) adopts the pluralist view that the construction of index numbers has to be adjusted to the purpose that the index number is supposed to serve. There is no

<sup>&</sup>lt;sup>7</sup>See (Salerno et al., 2020) and (Israel, 2022) who identify and discuss an inconsistency in Carl Menger's monetary theory regarding his use of the inner and outer exchange value of money.

general price level that can be captured in one index alone that serves all purposes. It is possible that one index provides a relatively good approximation to serve many purposes, but that would be "a statistical coincidence" not an "essential necessity." The three main purposes that Haberler identifies are (1) the comparison of income over time and place, (2) providing a standard of deferred payment, and (3) providing a guideline to monetary policy. He devotes one chapter to each of these purposes and then closes with a short final chapter on the price index in business cycle theory.

#### Income comparisons

Haberler begins by distinguishing three notions of income: nominal income, income in kind (*Naturaleinkommen*) and psychic income. Nominal income is just monetary income. Income in kind corresponds to the bundle of real goods and services that are bought for it. And psychic income is what really matters (Fetter, 1915). It refers to the subjective want satisfaction that we get out of consuming the income in kind.

From the outset, it is acknowledged that an index number cannot measure psychic income. "This is not possible because we cannot tell from the goods we consume and their prices what sensations of pleasure they evoke, what needs they satisfy" (Haberler, 1927, p. 81). Psychic income depends not only on nominal income and money prices, but also on the subjective level of needs. But Haberler also emphasizes that even if we assume needs to remain constant, from one period to the next, we still could not measure and compare psychic income objectively. Haberler points out that every shift in prices causes changes in the consumption pattern, and even with constant needs, we could not know, if the satisfaction of one need that is prevented by a price increase for one product is outweighed by the satisfaction made possible by the decline of another price or by a new product that enters the market (p. 81).

Money provides, of course, a homogeneous unit to measure nominal income. It is thus relatively easy to assess, although Haberler concedes that, even here, it can sometimes be difficult to clearly distinguish between income and wealth in practice (p. 77). He then discusses the possibility of comparing the "real income" of one and the same person in two different periods, and he notes that neither the Paasche nor the Laspeyres index seem to hit the nail on the head. He asks: "What is actually supposed to be compared?" (p. 80). What we want to compare is the "real income that is covered by a given sum of money in period 1, with the real income that the same sum of money can get in period 2, that is, two *actual* amounts [*Istbeträge*]" (p. 80). However, both the Laspeyres and the Paasche index put actual amounts in relation to "fictitious amounts." The denominator of the Laspeyres index refers to an actual amount. It is the sum of money  $\sum p_1q_1$  actually spent on the bundle of goods  $\sum q_1$ .<sup>8</sup> The enumerator of the Laspeyres index  $\sum p_2q_1$  "is a fictitious amount, because neither is the bundle of goods  $\sum q_1$  bought in period 2, but the bundle  $\sum q_2$ , nor is the sum of money  $\sum p_2q_1$  spent, but the sum  $\sum p_2q_2$ ." A similar reasoning applies to the Paasche index, where the denominator is a fictitious amount.

The "real income that one has in mind is neither  $\sum q_1$  nor  $\sum q_2$  and even less  $\sum \frac{q_1+q_2}{2}$ or  $\sum \sqrt{q_1q_2}$ , but is something that stands behind the income in kind" (p. 80). The income in kind is only its external form. It consists of different compositions,  $\sum q_1$  and  $\sum q_2$ , from which one cannot directly infer which composition comprises the higher real income. But if there is "a common something" to the incomes in kind by which they can be compared, it is the underlying real income, which Haberler prefers to call psychic income. We can get a handle on it through the observable incomes in kind only by making numerous assumptions. Both the Laspevres and Paasche index can then be meaningfully interpreted.

Haberler thus returns to the "crux of the doctrine of index numbers" already mentioned in the first part of the book: should one use the quantities of goods from the first period or the second? Should one use the formula of Laspeyres or that of Paasche? It turns out that, just as Fisher suggested on purely statistical grounds, a compromise between the two seems most appropriate. Haberler was the first to develop an argument for such a compromise

<sup>&</sup>lt;sup>8</sup>We continue to use Haberler's notation, although it is misleading to refer to a bundle of commodities as the sum of the quantities of these commodities. A vector of quantities would be more appropriate.

rooted in basic economic theory. We will look at it in detail in Section 3.

#### A standard of deferred payment

Haberler's discussion of price indices as possible standards of deferred payment is covered very briefly in Chapter IV of Part II (pp. 100-102). By referring to Mises (1953), he points out that especially the Anglo-American literature has considered the problem of changes in the price level "almost exclusively" from the vantage point of credit transactions. The question is whether the amount of the credit should be adjusted so as to correct for any changes in the purchasing power of money. In this way neither creditor nor debtor benefit at the expense of the other. But what index should be used?

A version of the income index, a compromise between Paasche and Laspeyres, could be used, but then an obvious problem emerges. In fact, there might be two indices to consider, instead of just one. There is not one objective solution but two subjective ones. The personal indices for creditor and debtor can be different, not because of a lack of data or calculation methods, but because of the nature of the problem. "The theoretician of index numbers is immediately ready to overcome the problem by constructing an average - the loophole of index number theory (*die Eselsbrücke der Indextheorie*)" (Haberler, 1927, p. 101). In such cases both parties to the credit transaction would carry half of the risk of their personal income indices diverging. Haberler emphasizes again that there simply is no way of scientifically building upon an economy-wide, objective purchasing power of money. There simply is no such thing in his view.

Now, compensating exactly for changes in the purchasing power of the sum of money lent and later payed back is typically not the goal that people have in mind when discussing this problem. Rather, the most frequent question is whether it would be better to society to benefit one of the two sides in a credit transaction at the expense of the other. Is it better to allow the class of debtors ("by which one typically thinks of entrepreneurs") or the class of creditors ("capitalists and rentiers") to benefit? Whatever the answer may be, in "normal times" the purchasing power of money does not change so much that any side of the transaction feels they were being taken advantage of by the other. And to make sure that times are normal is the task of monetary policy, which Haberler turns to next.

#### A guideline for monetary policy

In common parlance monetary policy is supposed to stabilize the price level, but it is not the "price level of Mister A or class B" but "the price level of the whole economy" (p. 103). In public opinion, Haberler points out, there is nothing problematic about this. It seems intuitively clear what is meant. "Not so for us. For us there are no economic phenomena that are not a complex web of individual economic actions; for us this way of speaking is therefore terribly unclear" and Haberler sets the goal of clarifying what could be meant exactly, when people say that "monetary policy should stabilize the price level."

He begins by briefly summarizing the old debates of the 1870s and 80s on bimetallism, which, according to him, have made first steps towards grasping the nature of the price level and the causes of its change. In particular, the distinction between causes that lie on the goods side and causes that lie on the money side is of great importance. Some advocates of gold tended to argue that the decreasing prices of the time were simply due to real economic growth. It is natural and desirable. Nothing needs to be done in terms of the money supply. Some advocates of bimetallism, on the other hand, thought that declining prices were caused by the demonetization of silver and a corresponding decline in the money stock. They argued for an increase in the money stock through the implementation of an international bimetallic standard. However, the debates have petered out as the international production of gold increased and prices stopped to fall.

Haberler (1927, pp. 109-110) criticizes both Menger and Mises for their use of the terms *inner* and *outer* exchange value as having muddled the waters of these historical debates. He devotes a lengthy footnote on criticizing the inconsistent use of the terms in Mises (1912).<sup>9</sup>

<sup>&</sup>lt;sup>9</sup>Menger elaborated on these terms in his last scientific publication on monetary theory (Menger, 1970, (1909)) and, indeed, ran into an inconsistency with his underlying theory of value and price (Israel, 2022).

These terms have led to distractions from some of the more useful notions that came out of the debate on bimetallism, such as esteem-value, labor-value or utility-value of money. These are different conceptions of the value of money, on the basis of which various stabilization proposals have been formulated.

Haberler argues that the notion of esteem value essentially captured what the inner exchange value of money is supposed to mean. He criticizes Mises for not having defined it properly, but he himself does not provide any clarifications on this matter. Just as Mises, he vaguely equates a change in the inner value or esteem-value of money to a change due to causes on the money side, such as a decrease in the volume of circulation. Stabilization policies based on the esteem-value would only try to counter changes caused by the money side.

The labor-value of money refers to the wage level, that is, how much labor services money buys. A labor standard would try to stabilize wage or income levels.<sup>10</sup> Lastly, the utility standard would attempt to stabilize the marginal utility of money, or the weighted subjective value of money. "How it is supposed to be measured is a question in itself" Haberler (1927, p. 111) states. He is doubtful about the practicality of all of these proposals, but what he wants to show is simply that the discussion about stabilization is much richer than most statisticians and even most economists notice. It has not generally been about stabilizing an overall price level.

Haberler then discusses the effects of a decreasing price level due to changes on the goods side. Technological progress that leads to growth in one sector, a corresponding increase in the quantity produced and a reduction of the price of the product has generally no harmful effects in his view. There are adjustments that are not completely frictionless, as one would say in modern terms, but many of these frictions "would not be prevented by a monetary policy that targets price level stability" (p. 114). The adjustments triggered in the process tend to compensate each other quite well. For example, if the demand for the product is

<sup>&</sup>lt;sup>10</sup>Haberler (1931) discusses the labor standard and its implication in a bit more detail.

inelastic - a term Haberler does not use, but clearly means - costumers will spend overall less money on the product after the price decrease, so that there will be more money available to be spend on other goods and services. This "obviously cannot be a motive for depression" (p. 114). Moreover, technological progress implies lower per unit costs. If demand is inelastic and output in the progressing sector increases only moderately, then it it is likely that factors of production are freed up. They can now be used in those other sectors that have higher consumer demand because of the money saved. By contrast, if consumer demand is elastic, the reverse is likely to happen, the progressing sector will pull factors of production away from other sectors that now have lower consumer demand. This redirection of production efforts is completely natural in a dynamic economy and the possible negative consequences of downward adjustments in prices are relatively easy to cope with in light of general economic progress and lower costs (p. 115).

As far as the relationship of creditors and debtors is concerned, there are no structural effects. Firms as debtors are not systematically harmed by the reduction in the price level. Firms operating in the progressing sector are not hit by surprise. They are implementing the progress. Other businesses are confronted either with increasing consumer demand and lower costs (factors freed up to be used by them), or with lower consumer demand and increasing costs (factors pulled away from them). So, they can gain or lose. That is the challenge any business faces. The potential loss is really "*lucrum cessans*" (lost profit). By contrast, price deflation through a decline in the money stock as opposed to economic progress would generate "damnum emergens" (direct loss) on the side of the debtors (p. 115).

Overall, Haberler concludes that the effects of economic progress, that is, intensive growth, are not comparable to those of a deflation understood as a decline in the money stock. He briefly mentions extensive growth, too, for example, through population growth or the integration of new territories into the monetary economy, and states that its effects are likely to be closer to those of monetary deflation, but he does not go into any details.<sup>11</sup>

 $<sup>^{11}</sup>$ This distinction between extensive and intensive growth is also discussed in his League of Nations report on index numbers (Haberler, 1931).

Haberler (pp. 116-117) argues that bank credit, as opposed to active monetary policy, has played an important role in accommodating economic progress and preventing the price level from falling too sharply. Any attempt to stabilize, be it the labor-value, esteem-value or even utility-value, would have to control bank credit expansion. He believes that this would have hampered economic progress. Without bank credit expansion "the upswing would not have been as strong as it was." Following Schumpeter (1983, (1926)), he even claims that inflationary bank credit expansion, too, could contribute to economic progress, but he gives no justification for this claim. He just mentions that Schumpeter's own argument is not entirely satisfactory.<sup>12</sup>

Now, given all these complexities and unsettled questions, one might be surprised to find that Haberler identifies some reasons for why the notion of a general price level can be meaningful, in spite of all subjectivity. The first reason is that we can "console ourselves with the thought that we are only dealing with relatively short intervals in monetary policy" (pp. 118-119), and over short intervals the numerous assumptions made in price level comparisons are more likely to be sufficiently close to the truth. His second point is the continuity of the economy (Beharrungsvermögen der Wirtschaft). Individual price indices for various social groups do not diverge too much. And individual prices for goods and services do not typically fluctuate wildly. Therefore we can often find good approximations. There is no guarantee, but in reference to Edgeworth (1925), Haberler invokes considerations of probability that play an important role in the construction of index numbers. The probable continuity and smooth evolution of economic life makes it possible for us to construct meaningful indices in spite of lacks and errors in our data. This is why, we can in certain cases speak of a general price level of the economy, but it is "based on a statistical coincidence and not rooted in the nature of things that we can attach meaning to the expression: change in the economy-wide price level" (p. 120).

From here, Haberler transitions to business cycle theory, which he deals with very briefly

<sup>&</sup>lt;sup>12</sup>On these and related issues, see Haberler (1937) for his synthesis of business cycle theories.

on the final pages of his monograph. His main proposition is that in business cycle theory price level changes are symptoms not causes (p. 121). Their assessment in the context of business cycle research is not an end in itself but merely a means to understand the nature of the cycle and ultimately tame it, if possible. He emphasizes the importance of disaggregation by claiming that "modern business cycle theory, whether it is of the statistical-inductive or the theoretical kind, does not work anymore with a general economy-wide price level, but only with special indices of certain particularly reactive prices in connection with other indices (stock market turnover, interest rates, money in circulation, etc.)" (pp. 125-126). In this context he gives a shout-out to his future employer by praising the "Harvard approach" (*die Harvardschule*) to business cycle research and prediction that "had become a role model" and in which "a general price index is now only used as a stopgap measure" (p. 122). Even though business cycle research still had, and probably still has, a long way to go, Haberler was sure that "the economy-wide price index [...] has played his hand in this area" (p. 123).

This, of course, does not imply that the economic theory of index numbers has nothing to offer. It can, as Haberler apply showed, be put to very productive use in the assessment of real incomes on the microeconomic level.

### **3** Paasche and Laspeyres as Lower and Upper Bounds

Let us turn to what is undoubtedly the most significant contribution of Haberler's thesis from the vantage point of index number theory. It is contained in Chapter 2 on income comparisons in the second part of the book (Haberler, 1927, pp. 77-99). Haberler makes it very clear from the outset that he believes such comparisons to be possible only in a very limited sense. Telling which one of two incomes in kind comprises the higher psychic income would involve a value judgment on the part of the scientist (p. 83). Statements of the kind "income x at time  $t_1$  is 'objectively, scientifically' 50% higher than income y at time  $t_2$ " cannot be justified. There is no scientific basis for them, not merely because of a lack of data, but because of the subjective nature of the problem.

In Haberler's view it is up to the individual actors to decide what they prefer, not the scientists. The actor's preferences must serve as a guide to estimating income changes as best as we can, but many assumptions have to be made along the way, because in most situations, the relevant preferences are not known. The bundle of goods and services that a consumer decides to buy in a given period is shown to be preferred over all other available bundles within the budget constraint for that period, but it is not shown to be preferred over that are simply not affordable.

In most cases it seems uncontroversial to assume, for example, that people would prefer a higher nominal income given the same money prices. It seems uncontroversial to assume that a higher nominal income comprises a higher psychic income given the same money prices, although this is of course not necessarily true. One might think of an ascetic person who prefers less income (p. 84). But there are still other implicit assumptions made. We must assume the *ceteris paribus* clause. Haberler mentions three aspects that are held constant in our minds but could and very often do change in the real world: 1.) the level to which needs are satisfied, 2.) preferences and needs themselves, and 3.) a whole set of side conditions that he sums up as climatic, social, cultural, and political conditions. They are not measurable and have to be assumed to be constant between regions and times for which income is compared. If we cannot reasonably assume them to be constant, there is no other way but to qualitatively compare these conditions and come to a subjective evaluation of what situation is preferable. Nominal income and money prices do of course still play a role, but they are not the only aspects that matter (p. 85).

Haberler puts forth another assumption very early in the chapter. He assumes that 4.) the entire nominal income is consumed. None of it is saved and no capital is built (p. 77). He does this mainly in order to avoid any complications involved in the exact separation of income and wealth. This assumption, he notes, corresponds best to ordinary wage workers.

He denotes the nominal incomes of period 1 and 2 as  $E_1$  and  $E_2$ , respectively. If all of it is spent on consumable goods and services, we can write  $E_1 = \sum p_1 q_1$  and  $E_2 = \sum p_2 q_2$ .

There are two more assumptions that come into play. Haberler points out that for a meaningful numerical comparison of incomes, it is necessary that: 5.) all goods are available in both periods or regions, and 6.) there are markets on which they can freely be bought and sold, and all market participants can in principle buy the goods at the quantities they prefer given their budget constraint. The second assumption is crucial and often neglected: income comparisons between regions and periods require markets based on voluntary exchange. If the incomes in kind,  $\sum q_1$  and  $\sum q_2$ , are not subject to consumer choice, such comparisons are meaningless.

Haberler's analysis from then on rests on thinking through the implications of consumer choices (*Wahlakte*) under these assumptions as a window into the preferences of the choosing individual. He starts by formulating the first two basic principles of income comparison:

- Under otherwise identical conditions and given the same prices, one prefers the higher nominal income, that is, the higher nominal income comprises the higher psychic income. (pp. 83-86)
- 2. Under otherwise identical conditions and given different prices, one prefers the nominal income of that period or region that covers the income in kind of the reference period or region and would even buy some more. (pp. 86-89)

The validity of both propositions can easily be seen. In fact, the first, as Haberler mentions, is so intuitive that it hardly needs any explanation. A higher nominal income under otherwise equal conditions, including equal prices, can always buy what the lower nominal income buys and more. It is therefore preferred. This reasoning is just extended to the case of unequal prices. If one nominal income under the given prices would suffice to buy the income in kind of the reference period or region (and a little bit more), it is preferred. It then comprises the higher psychic income. This sort of reasoning can be formalized and pushed further.

We take, for example, the bundle of goods chosen in the first period, i.e. the income in kind,  $\sum q_1$ , and determine the amount of money needed to buy that bundle at the prices of the second period,  $\sum p_2q_1$ . If the nominal income of the second period is higher, that is,  $E_2 > \sum p_2q_1$ , the income in kind chosen in the second period,  $\sum q_2$ , is preferred over the income in kind of the first period,  $\sum q_2$ , because if that was not the case, the consumer could have bought the same goods as in the first period. Indeed, the "could have" depends on the freedom to choose. It presumes markets, or more precisely "free consumer choice" (p. 89). Restrictions on trade hamper our ability to engage in income comparisons.

There are more implications that can be drawn from the second principle. If indeed the nominal income of period 2 exceeds the amount of money needed to buy the chosen bundle of period 1 at prices of period 2, that is,  $E_2 > \sum p_2 q_1$ , then, because  $E_1 = \sum p_1 q_1$  as all of the income is consumed, we can rewrite our inequality as:

$$\frac{E_2}{E_1} > \frac{\sum p_2 q_1}{\sum p_1 q_1}.$$

If nominal income has not changed, i.e.  $E_1 = E_2$ , the condition can be written as:

$$1 > \frac{\sum p_2 q_1}{\sum p_1 q_1},$$

where the second term corresponds to the Laspeyres index, for short  $\mathcal{L}_I$ . So, if the Laspeyres index is smaller than 1 and thus indicates a decrease in the price level, the same nominal income would be preferred at the prices of period 2. In other words, the price level for the respective person, under the above assumptions, has indeed decreased. The same income can now buy a better bundle of goods in period 2. The Laspeyres index not only tells us that the price level has indeed decreased, but also tells us by how much the price level has decreased at least. It gives us the upper bound of the rate of decrease.

If for example  $\mathcal{L}_I = 0.5$ , then half the income of period 1, i.e.  $E_2 = E_1/2$ , would be

preferred at prices of period 2 over the nominal income of period 1,  $E_1$ , at prices of period 1. It is also possible that an income that is even less than half the income of period 1 would be preferred at the prices of period 2. This can be the case if certain goods decrease so much in price that a certain bundle of goods becomes now affordable in period 2, even with less than half the nominal income of period 1, which was not affordable before but is preferred to the chosen bundle of period 1.

In general, the Laypeyres index indicates by how much nominal income has to change so as to be able to afford at least the income in kind of period 1. This is what one gets at least with a nominal income adjusted in accordance with  $\mathcal{L}_I$ . But it is possible, depending on the changes in the price structure, that one could buy a better bundle of goods and services that was not affordable before. Hence, Haberler (1927, p. 90) concludes:

#### 3. The Laspeyres index gives the upper bound for the subjective price level change.

If  $\mathcal{L}_I = 1.05$ , the price level has increased by at most 5%, in the sense that a nominal income in period 2 that is 5% bigger than that of period 1 buys at least as good a bundle than that chosen in period 1. It potentially buys a better one, and hence, the actual individual price level for the economic actor under consideration might have increased by less than 5%. Haberler emphasizes again that this conclusion only holds under otherwise equal conditions or if the changes in the conditions do not matter for consumer choice and welfare.

The same reasoning can be extended to the Paasche index, for short  $\mathcal{P}_I$ . We take the bundle of goods and services bought in period 2 and calculate the amount of money needed to buy this bundle at the prices of period 1,  $\sum p_1q_2$ . If the nominal income of period 1 fully covers this amount,  $E_1 > \sum p_1q_2$ , the bundle bought in period 1 is preferred over the bundle bought in period 2, since one could have bought the bundle chosen in period 2 already in period 1. Since  $E_2 = \sum p_2q_2$ , we can rewrite the previous inequality as

$$\frac{E_1}{E_2} > \frac{\sum p_1 q_2}{\sum p_2 q_2},$$

and through inversion we obtain:

$$\frac{E_2}{E_1} < \frac{\sum p_2 q_2}{\sum p_1 q_2} = \mathcal{P}_I$$

If nominal incomes are equal,  $E_1 = E_2$ , the inequality turns to  $1 < \mathcal{P}_I$ . So, if the Paasche index is bigger than 1 and thus indicates a price increase, we can say that at constant nominal income the prices of period 1 are preferred over the prices of period 2, and therefore the subjective price level has indeed increased. The price level has increased by at least the percentage indicated by the Paasche index. If, for example,  $1.05 = \mathcal{P}_I$ , then the price level has increased by at least 5% in the sense that a 5% increase in nominal income does still not guarantee that the consumer acquires a bundle at least as good as that of the first period.

Haberler (1927, p. 91) uses a clever pedagogical trick to illustrate the argument that the Paasche index constitutes the lower bound. Since the Paasche index is taking the income in kind of the second period,  $\sum q_2$ , as the benchmark, it can get confusing to think backwards in time.<sup>13</sup> Instead of taking an example of an income comparison over time, Haberler therefore gives the following example of an income comparison between regions.

We look at workers from Germany and England. Let 1 denote Germany and 2 denote England. If the Laspeyres index indicates a price decline, let us say,  $\mathcal{L}_I = \sum p_2 q_1 / \sum p_1 q_1 =$ 0.9, then we can say that the real consumption of the German worker would cost 10% less in England. In other words, a German worker could live as well in England on a 10% lower income as he does in Germany, potentially even better. The price level in England is *at least* 10% lower in England. If  $\mathcal{L}_I = \sum p_2 q_1 / \sum p_1 q_1 = 1.1$ , the German worker would live as well in England with a 10% higher income as he does in Germany. The price level in England is *at most* 10% higher.

Now, we can take the English perspective by looking at the Paasche index. We ask ourselves what the English worker's real consumption,  $\sum q_2$ , costs at German prices and

<sup>&</sup>lt;sup>13</sup>Haberler mentions Weiss (1910) who argued that the Paasche index is preferable because it takes account of the most up-to-date bundle of consumer goods and services and rightly rejects this argument.

relate that to the costs of his consumption at home. Let us assume that  $\sum p_2 q_2 > \sum p_1 q_2$ , that is, the English worker's consumption bundle costs more in England than in Germany. To live in England is definitely more expansive than to live in Germany. This is reflected in the Paasche index as we can easily see by rearranging the inequality to

$$\mathcal{P}_I = \frac{\sum p_2 q_2}{\sum p_1 q_2} > 1$$

The Paasche index is above 1. So we can say that the price level is higher in England than in Germany, or since we have denoted England by 2 and Germany by 1, we could also say that the price level has increased from 1 to 2. Hence, given equal nominal income, if  $\mathcal{P}_I > 1$ , one would prefer to live in country 1 (or period 1). The price level has definitely increased from 1 to 2. If, for example,  $\mathcal{P}_I = \sum p_2 q_2 / \sum p_1 q_2 = 2$ , then even an income  $E_1$  that is half as big as  $E_2 = \sum p_2 q_2$  could buy as good a bundle as that which is chosen in 2,  $\sum q_2$ , but potentially a better one. The income  $E_1$  could potentially be even less than half of  $E_2$  and it would still be preferred under the prices of 1. The price level has thus *at least* doubled.

If the Paasche index falls, it does not imply that the price level has necessarily fallen, but it provides a lower bound. Let us say  $\mathcal{P}_I = \sum p_2 q_2 / \sum p_1 q_2 = 0.5$ , then the bundle chosen in 2 would cost twice as much at the prices of 1. So if  $E_1$  is twice as high as  $E_2$ , one would prefer to be in 1, that is, there was *at most* a doubling of the price level from 2 to 1 (backward in time), which means that there was *at most* a decrease of 50% from 1 to 2 (forward in time). This brings Haberler to his second main conclusion:

#### 4. The Paasche index gives the lower bound for the subjective price level change.

Haberler continues to discuss various cases of how the calculated values of  $\mathcal{L}_I$  and  $\mathcal{P}_I$ could relate to one another, and he admits that it is possible that  $\mathcal{L}_I < \mathcal{P}_I$ . This would lead to a contradiction. There would then exist an interval of values for  $E_2$  at which period 2 (country 2) would be preferred and not preferred at the same time. Haberler argues that this contradiction can be resolved by assuming changes in preferences, needs or conditions. But more interestingly, he shows that the apparent contradiction can also be explained by significant differences in income levels from one period or country to another (p. 96). As he argues, given changes in prices can imply a decrease in the price level for a person with low nominal income, but an increase in the price level for the same person if he had a high nominal income. So, there are, on top of the strong assumptions mentioned earlier, many further potential complications that might not be of the highest relevance in practice, but that still need to be pointed out "to show how complicated things are, which most index statisticians would never dream of" (p. 96)

However, Haberler also points out that such cases tend to be rare in practice. Typically, one obtains  $\mathcal{L}_I > \mathcal{P}_I$ . This is for a good reason, as Haberler argues by referring the the following slightly rearranged formulas of the two indices that we have seen in Section 3.1:

$$\mathcal{L}_{I} = \sum \frac{p_{1}q_{1}}{(\sum p_{1}q_{1})} \frac{p_{2}}{p_{1}}; \quad \mathcal{P}_{I} = \sum \frac{p_{1}q_{2}}{(\sum p_{1}q_{2})} \frac{p_{2}}{p_{1}}$$

This representation makes clear why the Paasche index tends to be lower than the Laspeyres index. Both indices sum up the weighted price ratios  $p_2/p_1$  over all goods and services. The Laspeyres index gives each ratio a weight of  $p_1q_1/\sum p_1q_1$ , corresponding to the expenditure share of the respective good in period 1. The Paasche index gives them a weight of  $p_1q_2/\sum p_1q_2$ . The only difference is that  $q_1$  is replaced by  $q_2$ . Because there is a negative correlation between prices and quantities, the higher the price increase  $p_2/p_1$  the more  $q_2$ tends to fall and vice versa. Goods with higher price increases therefore tend to receive lower weights in the Paasche index and higher weights in the Laspeyres index.<sup>14</sup> Hence, the Laspeyres index tends to be higher. In other words, the Paasche index takes substitutions in consumer choice, due to relative price changes, into account.

Haberler also points out that the two indices usually are relatively close to one another. Under the assumptions spelled out above, they set the range within which the true value lies,

<sup>&</sup>lt;sup>14</sup>One can ignore the denominators in the weights as they usually do not radically change. They correspond to overall expenditure.

but there is no way of knowing more exactly where it is. His analysis provides a justification for taking some average between the two indices, but, according to him, there is no way of objectively proving one average to be superior to others. He thinks that "in light of this realization, the dispute over the right compromise formula seems almost laughable" (p. 94), but it is a dispute that has not been settled to this day (Armknecht and Silver, 2012).

Haberler (1927, p. 94) summarizes his contribution to the field as follows: "We have thus succeeded in giving the compromise formulas a rational, factual justification, which was completely lacking until now." The argument presented above is not only a significant contribution to the theory of index numbers. Haberler's mode of argumentation had a much broader and hitherto underappreciated impact on the further development of consumer choice theory.

### 4 A Blueprint for Revealed Preference Theory

The fact that Gottfried Haberler's habilitation thesis has never reached a non-German speaking audience may explain why nobody has ever explicitly shown how his analysis published in 1927 provides a showcase example for arguing within what less than two decades later became known as the theory of revealed preferences. Gottfried Haberler provides the blueprint for Paul Samuelson who was among his first students when he joined the Harvard faculty in 1936. Only two years after taking his first class with Haberler, Samuelson (1938) publishes his seminal paper in *Economica* which sets the foundations for revealed preference theory. It is possible that he never read the *Meaning of Index Numbers*.<sup>15</sup> He does not cite it in any of his papers.

Even if Samuelson has never read the book, he was in his early twenties when he learnt from Haberler through his lectures and seminars. As a brilliant and ambitious young student he must have absorbed his teacher's way of arguing. His praising words on Haberler cited in

<sup>&</sup>lt;sup>15</sup>At least as a child "Paul resisted learning other languages" according to Backhouse (2017, p. 11). This of course might have changed later in life. As of October 25, 1930, reading knowledge in both French and German was a requirement for obtaining an economics graduate degree from Harvard (Collier, 2015).

the introduction suggest as much. In fact, the connection between Haberler's contributions to index number theory and the notion of revealed preference was admitted by Samuelson many years later. Upon reflecting on the origins of his *Foundations of Economic Analysis*<sup>16</sup> he briefly discussed the revealed preference approach and wrote: "But I did not in 1938 'refine' that: as often mentioned, my approach arose from considering Haberlerian restrictions that apply to Laspeyres and Paasche quantity index numbers for a 'rational consumer' [...]" (Samuelson, 1998, p. 1381).

That it is "often mentioned" seems exaggerated, although it has been mentioned. Samuelson himself refers in passing to Haberler in 1950, during the second wave of publications on revealed preferences, but it is only an anecdotal reference with a somewhat self-serving tone.<sup>17</sup> It does not reveal the close proximity between the arguments. The secondary literature also only tends to refer to a connection of some kind, but does not explicitly describe it. Backhouse (2017, pp. 180-182, 203) refers to the same anecdote and mentions Haberler's habilitation thesis (as doctoral thesis), but his account of Haberler's analysis rests very broad and too imprecise to see how much of the revealed preference paradigm is already contained in his argument.<sup>18</sup> Hands (2014, p. 97) also quotes Samuelson's anecdote, but

 $^{17}$ Samuelson (1950, p. 369) states:

My own work in this direction grew out of a remark made to me by Professor Haberler in his I936 international trade seminar at Harvard. "How do you know indifference curves are concave?" My quick retort was "Well, if they're not, your whole theory of index numbers is worthless." Later I got to thinking about the implications of this answer (disregarding the fact that it is not worded quite accurately). Being then full of Professor Leontief's analysis of indifference curves, I suddenly realised that we could dispense with almost all notions of utility: starting from a few logical axioms of demand consistency, I could derive the whole of the valid utility analysis as corollaries.

It seems as if Haberler and/or Samuelson meant to say/write "convex" instead of "concave." Standard indifference curves are convex to the origin. Samuelson's challenge was to show that he can derive the convexity of indifference curves from the principle of revealed preference.

<sup>18</sup>Backhouse (2017, ch. 9) provides a very informative account of the relationship between Haberler and Samuelson. It is undeniable that Haberler played a formative role in Samuelson's economic education. Backhouse (2017, p. 170) writes:

Samuelson praised Haberler for being consistent and yet eclectic, cautious and yet open to new ideas. Comparing him with two other Austrian economists, he noted that he was more

 $<sup>^{16}</sup>$ As has been pointed out by Wong (2006, p. 67) and Hands (2014, p. 102), the term revealed preference does not play a big role in Samuelson's doctoral dissertation which is the *Foundations of Economic Analysis* (Samuelson, 1983), but it appears in the subsection on index numbers. The reference to Haberler is only in passing as one of the contributors to index number theory.

his analysis focuses on different aspects of the history and evolution of revealed preference theory. Weintraub (1983) highlights a close similarity to Abraham Wald's work of the mid 1930s, but Haberler's contributions are understandably not discussed as they do not fit into the theme of the paper. Overall, it seems fair to say that the connection to Haberler's work is underappreciated. But if one takes a close look at Samuelson's papers, a striking resemblance appears.

Samuelson's 1938 Note on the Pure Theory of Consumer's Behaviour starts out by proposing a more direct approach to consumer choice, using fewer assumptions concerning utility. Ultimately, he wanted to reject both cardinal and ordinal utility. He is "willing to dispense with the utility concept and its vestigial remnants" (p. 68). The whole point of Samuelson's article, as he stated ten years later, was to show "that the economic theory of consumer's behaviour can be largely built up on the notion of 'revealed preference'." This means that by

comparing the costs of different combinations of goods at different relative price situations, we can infer whether a given batch of goods is preferred to another batch; the individual guinea-pig, by his market behaviour, reveals his preference pattern - if there is such a consistent pattern. (Samuelson, 1948, p. 243)

Haberler's demonstration that the Paasche and Laspeyres indices, under certain assumptions, correspond to the lower and upper bound within which the actual subjective price level lies, is built on exactly the same premise. Interestingly, both in 1938 and in 1948, Samuelson refers directly to index number theory. In 1938, Samuelson suggests "the consideration of matters lying close to the modern theory of index numbers" (p. 64). And in 1948, he explains his basic idea as follows:

The central notion underlying the theory of revealed preference, and indeed the whole modern economic theory of index numbers, is very simple. Through any

thoughtful than Ludwig von Mises, who "exploded rather than ruminated," or Schumpeter, who "sparkled," throwing out ideas and criticisms.

observed equilibrium point, A, draw the budget-equation straight line with arithmetical slope given by the observed price ratio. Then all combinations of goods on or within the budget line could have been bought in preference to what was actually bought. But they weren't. Hence, they are all "revealed" to be inferior to A. No other line of reasoning is needed. (Samuelson, 1948, p. 244 (emphasis added))

Instead of making assumptions about utility, the notion of revealed preference provides "operational meaningful foundations" for consumer theory. Instead of postulating preferences to derive choices, one takes choices to derive constraints on preferences and demand. Choice is put at center stage and becomes the starting point of analysis. This is precisely what should be done according to Haberler:

We now ask: who decides which income in kind encompasses the greater psychic income, i.e. which provides the greater satisfaction? Is it science? No! The economic subjects prefer the one to the other, the economic subjects decide which income in kind they prefer, and that which the subjects prefer is called by theory the greater real income; of the income in kind which the subjects have chosen, theory says that it is the one which provides the greater psychic income. (Haberler, 1927, p. 82)

This is the premise: individuals (or guinea-pigs) choose. Their choice becomes the benchmark. If their choices are "consistent" as Samuelson called it, which essentially means that they satisfy the assumptions that Haberler outlined in detail, the conclusions follow. To recap, Haberler refers explicitly to 6 assumptions:

- 1. The level to which needs are satisfied is held constant.
- 2. Preferences and needs themselves are held constant.

- 3. A whole range of side conditions, summed up as climatic, social, cultural, political conditions, are held constant.
- 4. The entire nominal income is consumed.
- 5. All goods are available in both periods (regions).
- 6. They can be bought freely on markets.

Samuelson would simply call it *rational*, *consistent* or *coherent* consumer behavior. He does not spell out what precisely that entails and thereby downplays the role of auxiliary assumptions. Would it not sometimes be rational for consumers to change their mind when conditions change? It seems so. Haberler's explicit assumptions provide a good overview of what needs to be satisfied to make the very first and essential step in Samuelson's analysis valid.

Samuelson (1938, pp. 64-65) stipulates two "positions" of an agent with different incomes, I and I', and price structures,  $p_i$  and  $p'_i$ , in which corresponding sets of consumer goods are bought,  $\psi_i$  and  $\psi'_i$ . In his notation, overall expenditure is given by:

$$[\psi p] = \sum_{i=1}^{n} \psi_i p_i,$$

and

$$[\psi' p'] = \sum_{i=1}^{n} \psi'_i p'_i.$$

The next step is to "combine the prices of the first position with the batch of goods bought in the second. The total cost of such a batch" is:

$$[\psi'p] = \sum_{i=1}^{n} \psi'_i p_i.$$

Samuelson (1938, p. 65) now just follows the same reasoning as Haberler in his index number theory:<sup>19</sup>

<sup>&</sup>lt;sup>19</sup>He switches between the terms "position", "situation" and "period" and between denoting the batch by

If this cost is less than or equal to the actual expenditure in the first period when the first batch of goods was actually bought, then it means that the individual could have purchased the second batch of goods with the price and income of the first situation, but did not choose to do so. That is, the first batch (x) [ $\psi$ ] was selected over (x') [ $\psi'$ ]. We may express this symbolically by saying

$$[\psi p'] \leq [\psi' p'] \Rightarrow (\psi) \prec (\psi')$$

The symbol  $\prec$  means that the batch  $(\psi')$  was chosen over the batch  $(\psi)$ , which could have been chosen but was not.  $(\psi')$  is preferred. Following the same reasoning, one also obtains:

$$[\psi'p] \le [\psi p] \Rightarrow (\psi') \prec (\psi)$$

Samuelson then introduces the "consistency" assumption, or postulate in his words, that  $(\psi) \prec (\psi')$  and  $(\psi') \prec (\psi)$  cannot both be true at the same time. From there he is capable of deriving what is today known as the *weak axiom of revealed preference*.

The very same basic idea that helped Haberler to show that the Paasche and Laspeyres indices can be interpreted as upper and lower bounds of the subjective price level change of a consumer, allowed Samuelson to set the stage for revealed preference theory. By doing so, (Samuelson, 1938, p. 71) hoped "that the orientation given here is more directly based upon those elements which must be taken as data by economic science."

### 5 Conclusion

Gottfried Haberler made many important contributions to economics. One of the lesser known contributions is contained in his 1927 habilitation thesis on the *Meaning of Index*  $\overline{(\psi)}$  and (x).

*Numbers.* Building on basic economic principles, Haberler has shown that the well-known Paasche and Laspeyres indices can be interpreted as the lower and upper bounds within which the true index value lies. He was the first to provide an economic, as opposed to a purely statistical, rationale for taking averages of the two standard indices, such as suggested by Irving Fisher (1922).

Haberler's analysis is rooted firmly in the subjective value tradition of the Austrian school. He warns that there is no such thing as an objective change in the price level under the normal circumstances of economic life. The best one can do is to compute suitable approximations that are sufficiently accurate to serve their respective purposes. Their accuracy, however, cannot be proven beyond any reasonable doubt in many situations. It is contingent on statistical coincidences, which Haberler regards at least under normal circumstances as relatively probable. But he emphasizes that there is nothing in the nature of the price level by which we can determine its rate of change objectively.

Haberler decidedly rejects monism and adopts a *pluralist* position. Since there is no one true price index that serves all purposes, different indices have to be constructed and carefully adjusted to the respective purposes they are supposed to serve. Mathematicalstatistical considerations are important, but the primary guide in this endeavor is economics. Only economics can uncover the underlying meaning of index numbers. Its primary lesson is caution. Sometimes it is advisable to just try to "illustrate" rather than "measure" the purchasing power of money (Walther, 1912, p. 2). Haberler (1927, p. 88) writes:

No answer to a question impresses the layman more than a number and no answer should be more distrusted by the scientist. To encourage this distrust is one of the main tasks of this book.

Haberler's argument for why the Paasche and Laspeyres indices can be considered bounds has been absorbed seamlessly into the body of economic knowledge, but its origins have been blurred. This might be due to the fact that his habilitation thesis has never been translated into English. When he joined the Harvard faculty in 1936, Haberler entered a vibrant research environment. His most famous student, Paul Samuelson (1998, p. 1376) has put it as follows: "New European blood - Schumpeter, Leontief, Gottfried Haberler - plus soon-to-come Alvin Hansen, the 'American Keynes,' was beginning to make Harvard the mecca for advanced economic research." It is well known that Haberler was very influential in Samuelson's development as an economists. The relationship and the appreciation that Samuelson had for his teacher Haberler is widely documented.

Although it has been mentioned at times that Haberler's index number theory is connected to Samuelson's theory of revealed preferences, the close resemblance between the two has been underappreciated. Haberler's argument provides a blueprint for for revealed preference theory. He could be considered the pioneer of revealed preference. This suggests an entirely unexpected influence of the Austrian school on one of the main representatives of modern neoclassical economics and one of his main contributions.

While revealed preference theory was subjected to quite severe criticism (Wong, 2006), it is undoubtedly the case that it had a major impact on the development of consumer demand theory by braking the emerging dominance of the theories of Pareto, Slutsky, Allen and Hicks based on ordinal utility in the 1930s (Moscati, 2007, p. 367). Samuelson won the Nobel Memorial Prize in economics in 1970 for "the scientific work through which he has developed static and dynamic economic theory and actively contributed to raising the level of analysis in economic science." His contributions to microeconomic consumption theory played a role in that. Samuelson (1992, p. 101) once said that Haberler would have qualified "for about two-and-a-half Nobel Prizes." One additional reason that could make this statement a bit less of an exaggeration was documented here.

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